A critical appraisal of asymptotic 3D-to-2D data transformation and the potential of complex frequency 2.5-D modelling in seismic full waveform inversion

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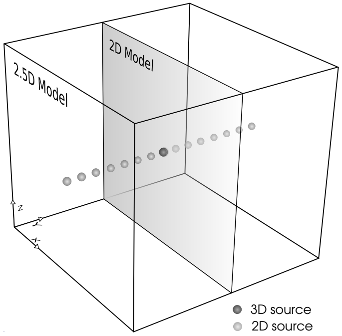
Motivation: The 3D-to-2D problem



Field data: point source ↔ Synthetics: line source

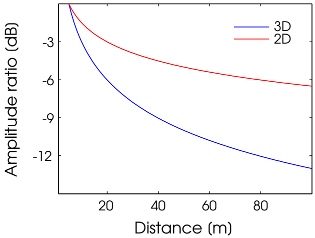
Outline and thesis objectives

1. Introduction: Line source and point source characteristics
2. Assessment of the validity of 3D-to-2D data transformation
3. An approach to frequency domain 2.5-D modelling
4. Summary and conclusions

Differences between 3-D and 2-D wave propagation

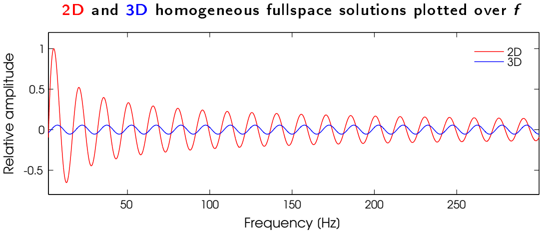
Differences in **amplitude decay behaviour**

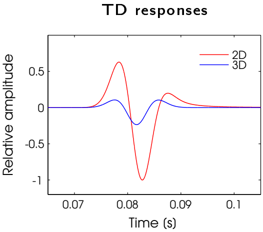
- Point source spreads energy over surface of a sphere
  $\rightarrow$ 3D amplitudes scale with $1/r$
- Line source spreads energy over surface of a cylinder
  $\rightarrow$ 2D amplitudes scale with $1/\sqrt{r}$



Differences between 3-D and 2-D wave propagation

Differences in **wavelet shape and spectral properties**





- 2D Green's function scales with frequency
- Solutions phase shifted by $\pi/4$ in the far field
- 2D wavelet is asymmetric and has a "long tail"

Theory and limitations of asymptotic filtering

Bleistein's filter in the time and the frequency domain

Filter is derived by forming the ratio $\bar{G}^{2D}/\bar{G}^{3D}$ between the 3D and the **asymptotic** 2D Green's function.

$$\bar{G}^{2D}(\omega) = \bar{G}^{3D}(\omega) \cdot exp\left(\frac{i\pi}{4}\right)\sqrt{\frac{2\pi\sigma}{|\omega|}}$$

$$G^{2D}(t) \approx \sqrt{t} \cdot \left[\frac{1}{\sqrt{t}} * G^{3D}(t)\right]$$

with $\sigma_{homog} = cr = c^2 t$ and $\sigma_{general} = \int_s c(s)ds \rightarrow$ Raytracing!
One individual scaling factor σ for each arrival!

Theory and limitations of asymptotic filtering

Fundamental and practical limitations

Critical issues

- Asymptotic approximation breaks down in the near field
- Amplitude adjustment is done in a time-sample manner
- Ray paths are curved when the velocity is a funct. of space
- Ray approach is inadequate for overlapping/interfering events

Numerical evaluation of asymptotic filter procedures

Testing environment

Testing procedure

1. Establish 2D models of varying degrees of complexity
2. Set up 3D models by repeating 2D models along the y-axis
3. Compute 2D and 3D synthetics with a viscoelastic FDTD code
4. Compare true 2D and filtered 3D data in the TD and the FD

Main finding: Filter ...

... works well in acoustic media like models with blocky or stochastic anomalies

... fails in elastic media when interference between P and S waves occurs

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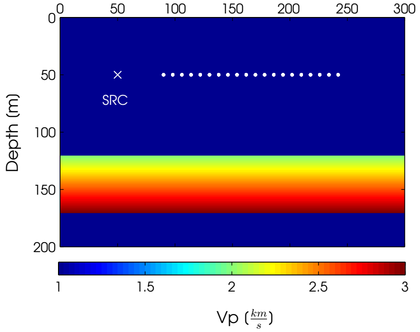
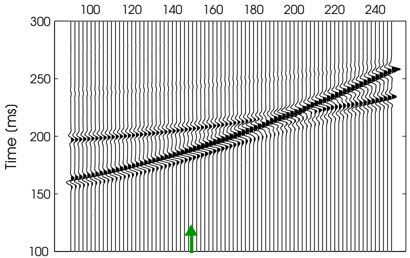
Example I: Acoustic fullspace with a velocity gradient
Model layout and synthetic seismic gather

Acoustic constant-density fullspace with a velocity gradient





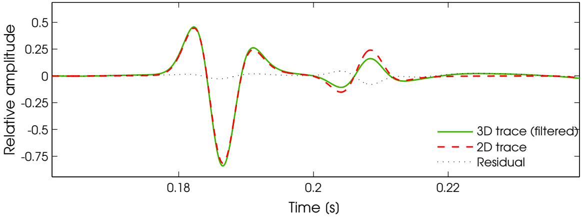
2D synthetic seismogram

100 Hz omnidirectional Ricker source

Example I: Acoustic fullspace with a velocity gradient

Sample trace in a distance of 150 m

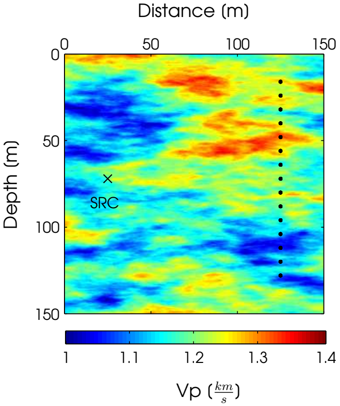
Sample trace at a distance of 150 m (time domain)

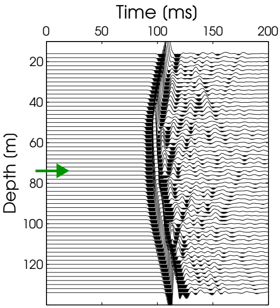


- 2nd arrival undercorrected by ∼ 30 %
- Mean RMS error ∼ 1.2 %
- Error accentuated where overlap occurs

Example II: Acoustic fullspace with stochastic fluctuations

Model layout and synthetic seismic gather





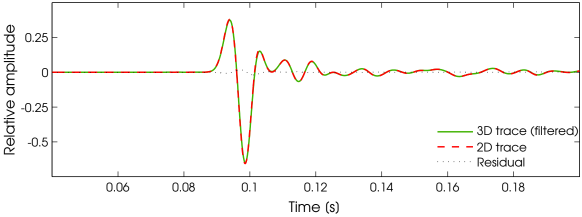
2D synthetic seismic gather

100 Hz omnidirectional Ricker source

Example II: Acoustic fullspace with stochastic fluctuations

Sample trace at a depth of 74 m

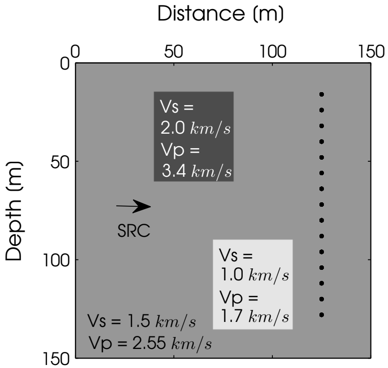
Sample trace at a depth of 74 m (time domain)



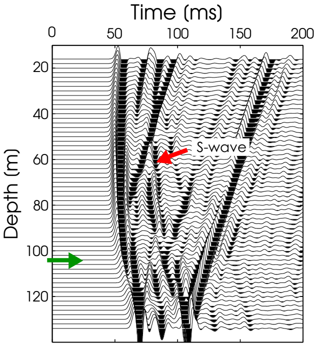
- Filter performs surprisingly well!
- Max. relative time domain error $\sim$ 2.0 %
- Mean RMS error $\sim$ 0.3 %

Example III: Elastic fullspace with block anomalies

Numerical appraisal of asymptotic filtering



$\rho = 2^3, 3^3, 4^3 kgm^{-3}$

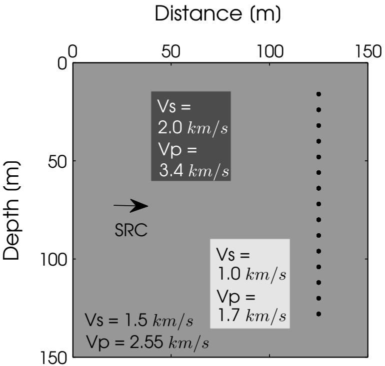


Particle velocity, x-component (2D)

100 Hz x-directed Ricker source

Example III: Elastic fullspace with block anomalies

Numerical appraisal of asymptotic filtering



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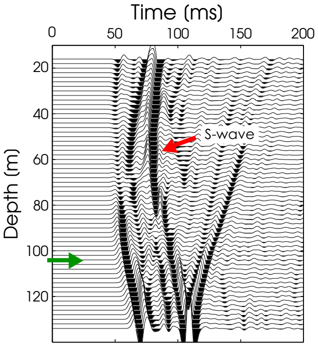


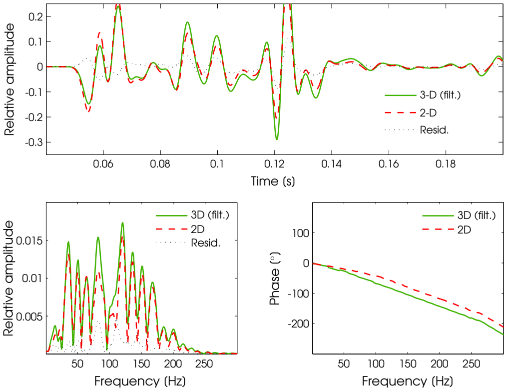
Particle velocity, z-component (2D)

100 Hz x-directed Ricker source

Example III: Elastic fullspace with block anomalies

Numerical appraisal of asymptotic filtering

Sample trace at a depth of 104 m



- Mean RMS error of $\sim$ 2.8 %
- Max. TD error of $\sim$ 36 %
- Significant phase error

Introduction to 2.5D modelling

Basic concept of 2.5-D modelling

- Based on a spatial Fourier transform of the 3D wave equation along the y-axis and solving it for N_{ky} wavenumbers
- Reduces memory requirements by breaking down the 3D problem to many 2D problems
- Can be performed in the time-wavenumber domain or in the frequency-wavenumber domain

Reduce N_{ky} to save computation time!

Introduction to 2.5D modelling

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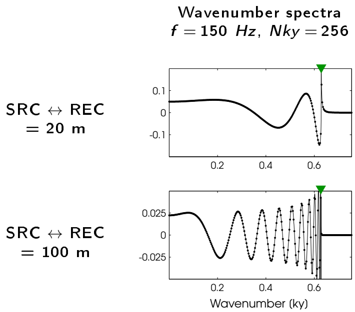
Wavenumber sampling issues in FD 2.5D modelling

- Degree of oscillation in k_y spectra increases with wavenumber value, SRC-REC distance and frequency
- Singularities or "poles" near certain critical wavenumber values complicate sampling

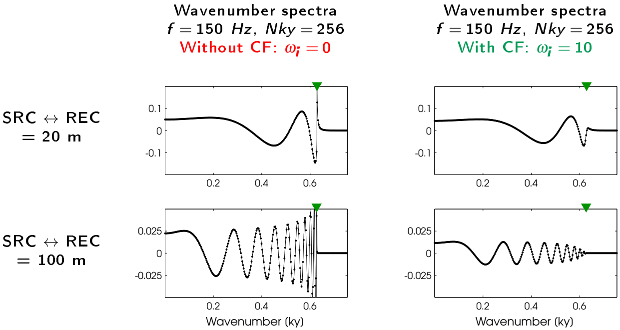


Note

k_y-aliasing or sampling close to a pole degrades FD solution

Mitigating the singularity problem with complex frequencies

- By introducing complex angular frequencies (i.e. $\omega = \omega_r + i\omega_i$) poles are "removed" from the k_y integration path
- Is equivalent to introducing time damping. Multiplying with $exp(\omega_i t)$ makes synthetics comparable to field data



2.5D FDFEM modelling with complex frequencies

2.5D complex frequency FEM modelling with complex frequencies: Procedure

1. Combined an acoustic FDFEM 2.5D forward solver with a complex frequency extension
2. Fixed the parameters ω_i at 10 and N_{ky} at 256
3. Performed 2.5D forward modelling on different acoustic models for a range of frequencies
4. Compared FEM 2.5D and FDM 3D data in time domain

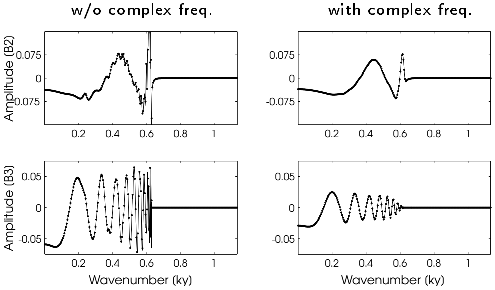
Numerical example: Acoustic fullspace with block anomalies

Wavenumber spectra before and after introducing complex frequencies

Acoustic block model, frequency $f = 150\ Hz$

SRC ↔ REC = 20 m

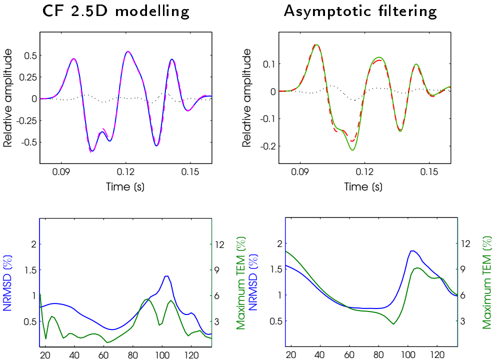
SRC ↔ REC = 100 m



Oscillations much better captured when frequency is made complex!

Numerical example: Acoustic fullspace with block anomalies

Sample trace 45: Comparison of CF 2.5D modelling and Asymptotic filtering



- No. of k_y samples reduced by factor of 10
- CF 2.5D modelling beats filtering in terms of errors!

Summary and conclusions

1. Asymptotic filtering
   - Asymptotic data transformation is suitable as long as the acoustic approximation is met
   - Acoustic full waveform inversions on 2D and filtered 3D indicate only marginal differences in the reconstructed model
   - An alternative to filtering is needed when a significant shear component is present and rays are strongly curved
2. 2.5D modelling
   - Complex frequencies stabilize wavenumber sampling and allows to reduce No. of k_y-samples by a factor of 10.
   - FEM 2.5D results compare very well to FDM 3D synthetics
   - Critical usefulness of CF 2.5D modelling expected in the elastic case

For Further Reading I

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Geophysical Prospecting. 34:686–703, 1986.

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Exploration Geophysics. 28:11–15, 1997.

For Further Reading II

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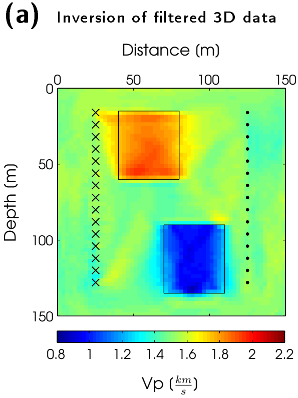
  Wavenumber Sampling Issues in 2.5D Frequency Domain Seismic Modelling.
  Pure and Applied Geophysics.: 2011.

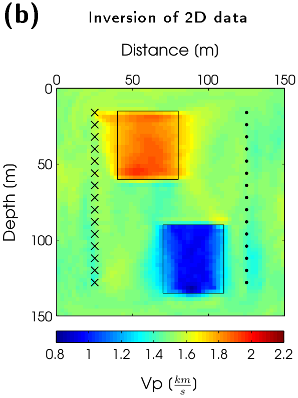
- J., Miksat, T.M., Müller and F., Wenzel.
  Simulating three-dimensional seismograms in 2.5-dimensional structures by combining two-dimensional finite-difference modelling and ray tracing.
  Geophysical Journal International. 174:309–315, 2008.

- P. R., Williamson and R. G., Pratt.
  A critical review of acoustic wave modeling procedures in 2.5 dimensions.
  Geophysics. 60:591–595, 1995.

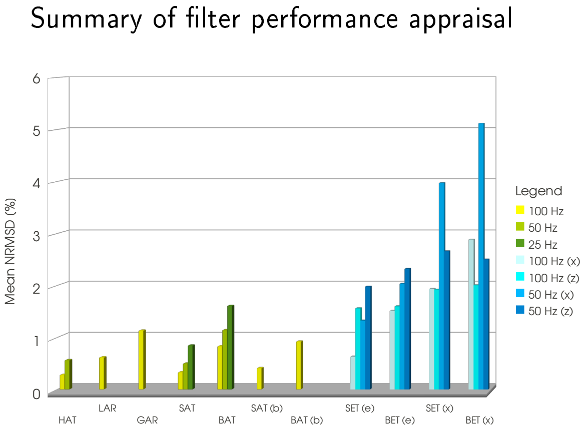
2D acoustic test-inversion of 2D and filtered 3D data

FD inversion using 9 frequencies b/w 20 and 260 *Hz* and 80 its.





Summary of filter performance appraisal



Differences between 3D and 2D wave propagation

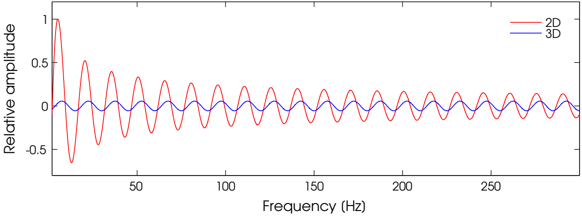
Differences in frequency-domain (FD): Green's function solutions for a homogeneous acoustic fullspace

3D Green's function in FD

$$\bar{G}^{3D}(\mathbf{r},\omega) = \frac{1}{4\pi r} exp(i\omega r/c)$$

2D Green's function in FD

$$\bar{G}^{2D}(\mathbf{r},\omega) = \frac{i}{4} H_0^{(1)}(\omega r/c)$$



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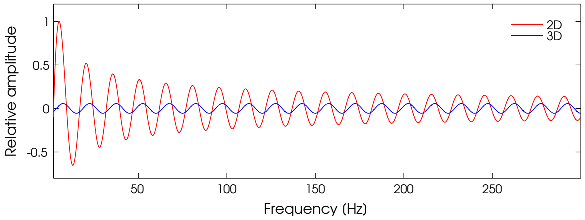
3D Green's function in FD

$$\bar{G}^{3D}(\mathbf{r},\omega) = \frac{1}{4\pi r} exp(i\omega r/c)$$

Asymptotic 2D GF in FD

$$\approx \frac{1}{2}\sqrt{\frac{c}{2\pi\omega r}} exp\left(\frac{i\omega r}{c}\right) exp\left(\frac{i\pi}{4}\right)$$



Differences between 3D and 2D wave propagation

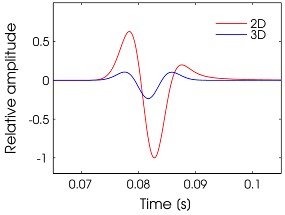
Differences in time-domain (TD): Green's function solutions for a homogeneous acoustic fullspace

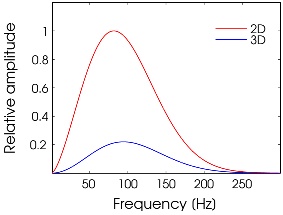
3D Green's function in TD

$$G^{3D}(\mathbf{r},t) = \frac{\delta(t - t_0 - r/c)}{4\pi r}$$

2D Green's function in TD

$$G^{2D}(\mathbf{r},t) = \frac{H(t - t_0 - r/c)}{2\pi\sqrt{(t-t_0)^2 - r^2/c^2}}$$

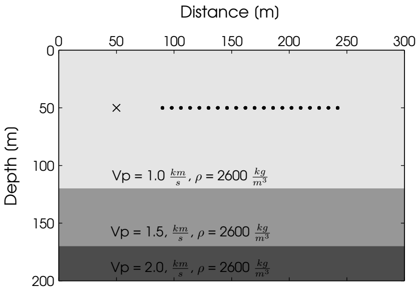


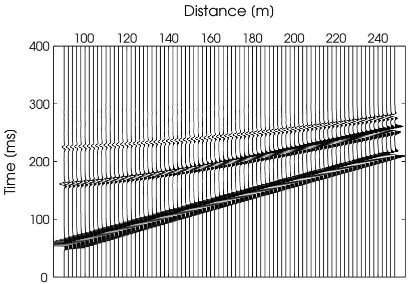


Example I: Layered acoustic fullspace

Model layout and synthetic seismic gather

Acoustic constant-density fullspace with 1D layering





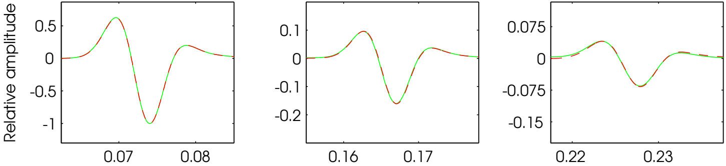
2D synthetic seismogram

100 Hz omnidirectional Ricker source

Example I: Layered acoustic fullspace

Sample trace: Comparison of filter implementations

Filtering in combination with raytracing



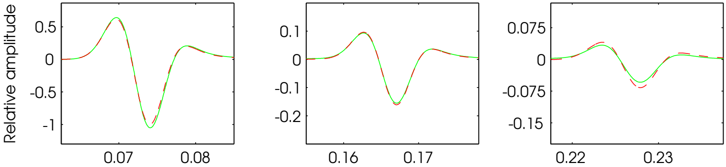
Relative amplitude error of ...

Event I	Event II	Event III
< 1 %	~ 1 %	~ 2 %

Example I: Layered acoustic fullspace

Sample trace: Comparison of filter implementations

Straight-ray approximate time-domain filtering

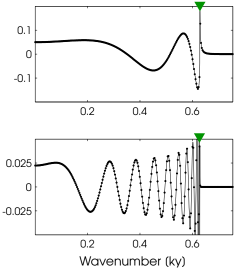


Relative amplitude error of ...

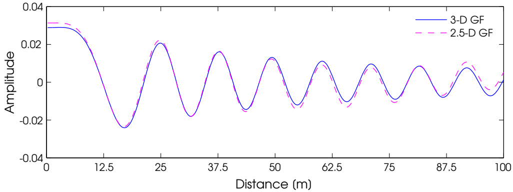
Event I	Event II	Event III
~ 5 %	~ 4 %	~ 20 %

Influence on the frequency domain solution

Wavenumber spectra
$f = 150\ Hz,\ Nky = 256$
Without CF: $\omega_i = 0$



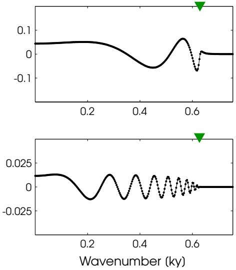
Frequency domain 2.5D (magenta) and 3D (blue) solutions



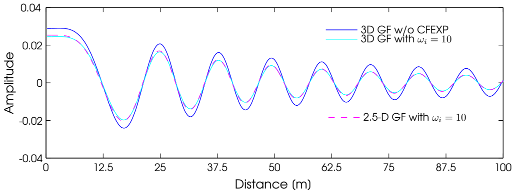
→Discrepancies between 2.5-D and 3D can be observed!

Influence on the frequency domain solution

Wavenumber spectra
$f = 150$ Hz, $Nky = 256$
With CF: $\omega_i = 10$



Frequency domain 2.5D (magenta) and 3D (blue) solutions

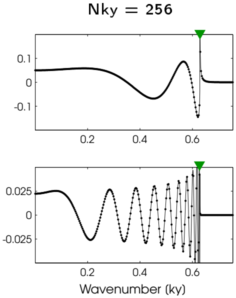


→Discrepancies between 2.5-D and 3D removed!

Origin of poles illustrated on acoustic GF's

Acoustic 2.5D fullspace solution $\tilde{G}^{2.5D}(k_y, \mathbf{r}, \omega) =$

$$\begin{cases} -\frac{1}{4}\left[J_0\left(r\sqrt{\frac{\omega^2}{c^2}-k_y^2}\right) - iY_0\left(r\sqrt{\frac{\omega^2}{c^2}-k_y^2}\right)\right] & , k_y < \frac{\omega}{c} \\ \frac{1}{2\pi}K_0\left(r\sqrt{k_y^2-\frac{\omega^2}{c^2}}\right) & , k_y > \frac{\omega}{c} \end{cases}$$



- When $w/c = k_y$ the squareroot becomes 0
- Y_0 and J_0 get singular when their argument is 0
- For $c = 1500\ m/s$ and $f = 150\ Hz$ a singularity is present at 0.6283

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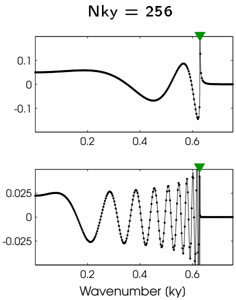

In complex media multiple poles contaminate the spectra and exacerbate wavenumber sampling!